ARJUNA

JEE

LECTURE NUMBER - 10

Basic Math

MSM SIR

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TABLE OF CONTENT

01 DPP DOUBTS (POPULAR)

02 PROPERTIES OF LOGARITHM

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DPP DOUBTS (POPULAR)

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DPP-2 (Q11)

11. Solution of $\frac{(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$

(1) $x \in (-\infty,-2] \cup \{1\}$

(2) $x \in (-\infty, -2]$

(3) $x \in (-\infty, -3] \cup \{1\}$

(4) $x \in (-\infty, -1] \cup \{2\}$

$x(-1)$

$$\frac{(x+2)(x-1)^2}{x^2-3x+4} \leq 0$$

NoNe, CB, AP

$x=1$

x^2-3x+4: AP, CB; $a=1$, $D=9-16<0$

$$x+2 \leq 0$$

$$x \in (-\infty,-2] \cup \{1\}$$

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DPP-3 (Q12)

12. $\dfrac{(2-x^2)(x-3)^3}{(x+1)(x^2-3x-4)} \geq 0$

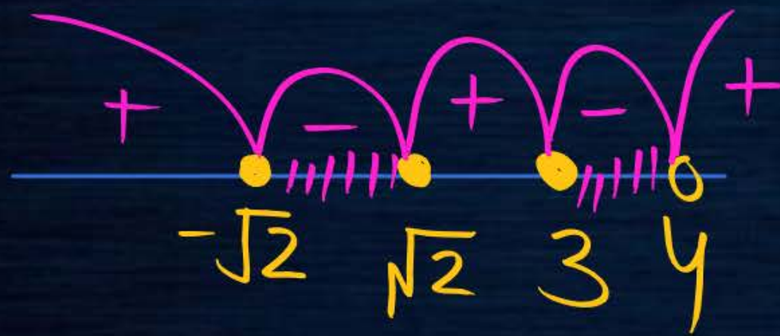
$\dfrac{(2-x^2)(x-3)}{(x+1)(x-4)(x+1)} \geq 0$

$\times(-1)$

$\dfrac{(x^2-2)(x-3)}{(x+1)^2(x-4)} \leq 0$

No-NE, CB, AP $\Rightarrow x \neq -1$

$\dfrac{(x-\sqrt{2})(x+\sqrt{2})(x-3)}{(x-4)} \leq 0$



$x \in \underbrace{[-\sqrt{2}, \sqrt{2}]} \cup [3,4) - \{-1\}$ Ans.

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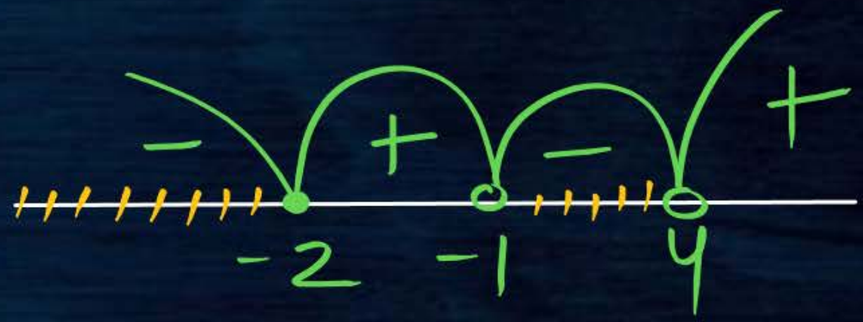
DPP-3 (Q14)

14. $\dfrac{(x+2)(x^2-2x+1)}{4+3x-x^2} \geq 0$

$\dfrac{(x+2)}{(x-4)(x+1)} \leq 0$



$x \in (-\infty, -2] \cup (-1, 4)$

Ans.

$\times (-1)$

$\dfrac{(x+2)(x-1)^2}{x^2-3x-4} \leq 0$

$\dfrac{(x+2)(x-1)^2}{(x-4)(x+1)} \leq 0$

No-Ne, CB, AP $\neq x=1$

DPP-3 (Q13)

$\sqrt{7} = 2.64$

$\sqrt{7}/2 = 1.32$

13. $\dfrac{5-4x}{3x^2-x-4} < 4$

$$\frac{5-4x}{3x^2-x-4} - 4 < 0$$

$$\frac{5-4x-12x^2+4x+16}{3x^2-x-4} < 0$$

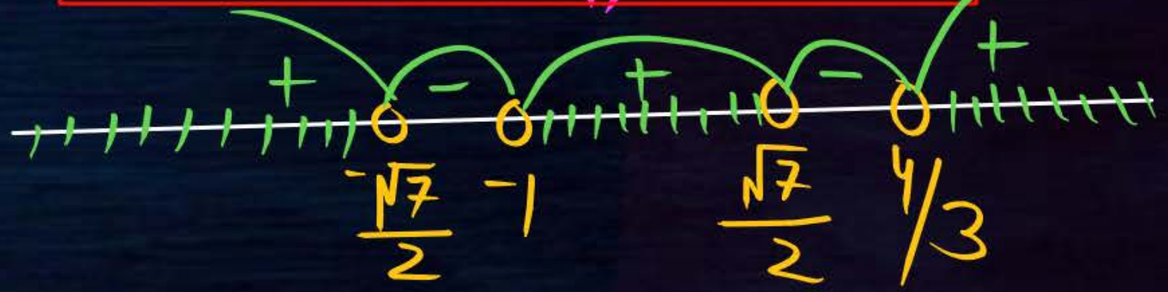
$$\frac{(-12x^2+21)}{3x^2-x-4} < 0$$

$\times(-1)$

$$\frac{12x^2-21}{3x^2-x-4} > 0$$

$$\frac{3(4x^2-7)}{3x^2+4x+3x-4} > 0$$

$$\frac{3(2x-\sqrt{7})(2x+\sqrt{7})}{(x+1)(3x-4)} > 0$$



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$$x \in \left(-\infty, -\frac{\sqrt{7}}{2}\right) \cup \left(-1, \frac{\sqrt{7}}{2}\right) \cup \left(\frac{4}{3}, \infty\right)$$

Ans.

DPP-3 (Q9)

APO, CB

9. $\dfrac{x^4+x^2+1}{x^2-4x-5}<0$

$x^4+x^2+1 \geq 1$

$x^4 \geq 0$, $x^2 \geq 0$

≥ 1

$x^{\text{Even}} \geq 0$

$\dfrac{1}{(x-5)(x+1)}<0$

$x \in (-1,5)$ Ans.

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(Learn)

$$(x^4+x^2+1) = (x^2-x+1)(x^2+x+1)$$

RHS $= (x^2-x+1)(x^2+x+1)$

$= x^4+x^3+x^2-x^3-x^2-x+x^2+x+1$

$= (x^4+x^2+1) =$ LHS.

Hence verified.

DPP-3 (Q9)

Method-2

9. $\dfrac{x^4+x^2+1}{x^2-4x-5}<0$

$$\frac{(x^2-x+1)(x^2+x+1)}{(x-5)(x+1)}<0$$

$a=1$, $D<0$, APo, CB

$a=1$, $D<0$, APo, CB

$$\frac{1}{(x-5)(x+1)}<0$$

$x\in(-1,5)$

Ans

DPP-5 (Q9)

9. Values of 'x' satisfying: $7 \le |x| + 4 < 10$

(1) $x \in [-3, 3]$

(2) $x \in [3, 6)$

(3) $x \in (-6, -3] \cup [3, 6)$

(4) $x \in (-6, -3]$

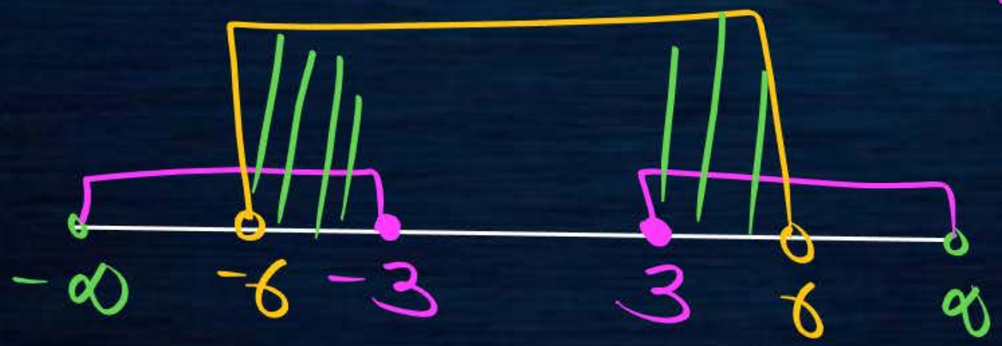
$7 - 4 \le |x| < 10 - 4$

$3 \le |x| < 6$

Double Inequality

$|x| \ge 3 \quad \cap \quad |x| < 6$

$x \in (-\infty, -3] \cup [3, \infty) \quad \cap \quad x \in (-6, 6)$



$x \in (-6, -3] \cup [3, 6)$

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DPP-5 (Q10)

10. If solution of inequality: $\frac{3-|x|}{|x|-5} > 0$ is $x \in (-b,-a) \cup (a,b)$, then $a+b=$ 8

(1) -2 (2) 2

(3) -5 (4) 8

$$\frac{3-|x|}{|x|-5} > 0$$

Case 1 $\Rightarrow$ $x \geq 0$ $\Rightarrow |x| = +x$

$$\times(-1)\left(\frac{3-x}{x-5} > 0\right)$$

$$\frac{x-3}{x-5} < 0$$

$x \in (3,5)$

$AC_1 \in (3,5)$ ✓

Case-2 $\Rightarrow$ $x < 0$ $\Rightarrow |x| = -x$

$$\frac{3-(-x)}{-x-5} > 0$$

$$\frac{3+x}{(-x-5)} > 0$$

$$\frac{3+x}{x+5} < 0$$

$x \in (-5,-3)$

$AC_2 \in (-5,-3)$ ✓

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Final ans →

$$x \in (-5, -3) \cup (3, 5)$$

OR

$$x \in (-b, -a) \cup (a, b)$$

$b = 5$, $a = 3$

$b = 5$ $\quad a = 3$

RECAP

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① $|a|+|b|=|a+b| \Rightarrow a \cdot b \geq 0$

② $|a|+|b|=|a-b| \Rightarrow a \cdot b \leq 0$

TWO IMPORTANT MODULUS RELATIONS

$$L = \log_a N \iff N = a^L$$

$$\cap \begin{cases} N > 0 \\ a > 0 \\ a \neq 1 \end{cases}$$

NOW LOGARITHM

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PROPERTIES OF LOGARITHM

★ ① $(\log_a N_1) + (\log_a N_2) = \log_a (N_1 \cdot N_2)$ (must Learn)

Same Base 'a'

ex) → • $(\log_2 3 + \log_2 6) = \log_2 (3 \times 6) = (\log_2 18)$

• $(\log_2 7 + \log_3 6) =$ Property cannot be applied

• $(\log_2 7 + \log_8 7) = \log_{2\times 8} 7$ (WRONG)

★ • $\log_2(7) + \log_2(8) = \log_2(7+8)$ (WRONG)

WRONG

WRONG

Right step

$\log_2(7 \times 8) = \log_2 56$ ✓

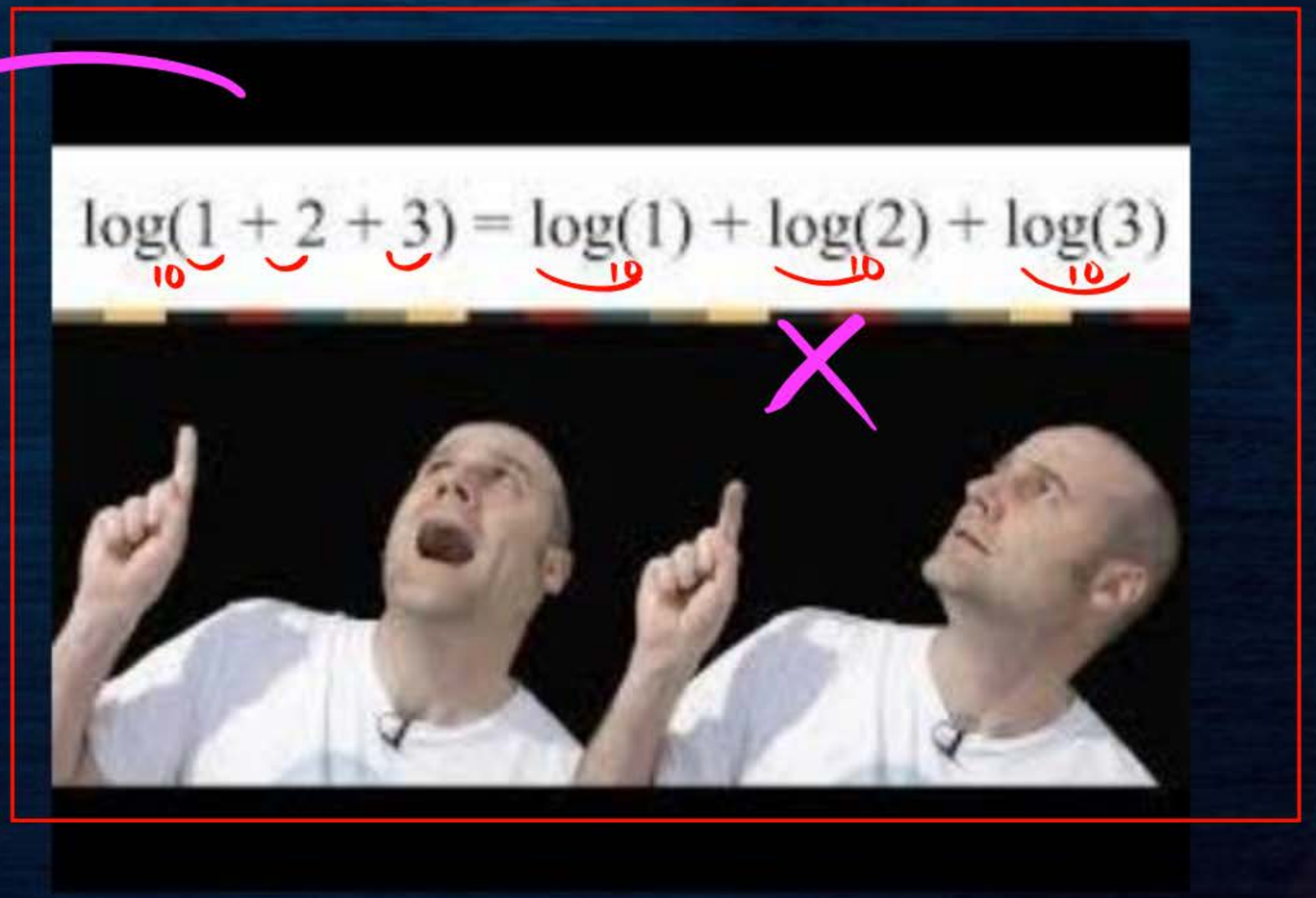
• $\log_2(7+8) = \log_2 7 + \log_2 8$ (WRONG)

Right step

✓ $\log_2 15$

RIGHT

$(\log_{10} 6)$ ✓



WRONG

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Proof (Not Important)

$\log_a N_1 = L_1 \Rightarrow \boxed{N_1 = a^{L_1}}$

$\log_a N_2 = L_2 \Rightarrow \boxed{N_2 = a^{L_2}}$

$\Rightarrow N_1 N_2 = a^{L_1} \times a^{L_2}$

$N_1 N_2 = a^{L_1 + L_2} = L$

$N_1 N_2 = N$

$\Rightarrow N = a^L$

$\Downarrow$

$L = \log_a N$

$L_1 + L_2 = \log_a N_1 N_2$

$\boxed{\log_a N_1 + \log_a N_2 = \log_a N_1 N_2}$

Hence Proved

② $\log_a N_1 - \log_a N_2 = \log_a\left(\frac{N_1}{N_2}\right)$

Base is same.

ex) • $\log_2 6 - \log_2 12$

$= \log_2\left(\frac{6}{12}\right)$ ✓

• $\log_6 2 - \log_{12} 2 \neq \log_{\frac{6}{12}} 2$

Proof (Not Imp)

$\log_a N_1 = L_1 \Rightarrow N_1 = a^{L_1}$

$\log_a N_2 = L_2 \Rightarrow N_2 = a^{L_2}$

Divide

$\frac{N_1}{N_2} = \frac{a^{L_1}}{a^{L_2}}$

$\frac{N_1}{N_2} = a^{L_1 - L_2} = a^L$

$N = a^L$

$L = \log_a N$

$L_1 - L_2 = \log_a \left(\frac{N_1}{N_2}\right)$

$\log_a N_1 - \log_a N_2 = \log_a \frac{N_1}{N_2}$

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$$\left(\log_a N_1 - \log_a N_2\right) \neq \log_a \frac{N_2}{N_1}$$

Uncommon mistake

③ ★ $\boxed{\left(\log_a b\right) = \left(\frac{\log_q b}{\log_q a}\right)}$

ex) $\left(\log_2 3 = \frac{\log_{20} 3}{\log_{20} 2}\right)$

PROOF → (NOT IMP)

RHS

$\log_q b = L_1 \Rightarrow \boxed{q^{L_1} = b}$ → Raised power $(1/L_1)$ → $(q^{L_1})^{\frac{1}{L_1}} = (b)^{\frac{1}{L_1}}$

$q^{L_1 \cdot \frac{1}{L_1}} = (b)^{\frac{1}{L_1}} = q$ ✓

$\log_q a = L_2 \Rightarrow \boxed{q^{L_2} = a}$ → Raised power $(\frac{1}{L_2})$ → $(q^{L_2})^{\frac{1}{L_2}} = (a)^{\frac{1}{L_2}}$

$\boxed{q = (a)^{\frac{1}{L_2}}}$ ✓

$$\alpha = (a)^{\frac{1}{L_2}} = (b)^{\frac{1}{L_1}}$$

Raised power "L_1"

$$(a^{\frac{1}{L_2}})^{L_1} = (b^{\frac{1}{L_1}})^{L_1}$$

$$a^{\frac{L_1}{L_2}} = b$$

L on $a^{\frac{L_1}{L_2}}$, N on b

$$N = a^L$$

$$\Downarrow$$

$$L = \log_a N \Longrightarrow \frac{(L_1)}{((L_2))} = \log_a b = \frac{(\log_\alpha b)}{((\log_\alpha a))}$$

Hence Proved.

$$\log_a b = \frac{\log b}{\log a} = \frac{\ln b}{\ln a}$$

(4) $\log_a a = 1$ ★

PROOF → $\frac{\log_q a}{\log_q a} = 1$

OR $\log_a a^N = L$

$N = a^L$

$a^1 = a^L$

$L = 1$

(5) $\log_a\left(\frac{1}{a}\right) = -1$

Learn

$\log_a N = L \xrightarrow{N=\frac{1}{a}} \log_a \frac{1}{a} = L$

$N = a^L$

$\frac{1}{a} = a^L \Rightarrow a^{-1} = a^L$

$\Rightarrow L = -1$

Ex)—

- $\log_2 2 = 1$
- $\log_\pi \pi = 1$
- $\log_3 3 = 1$
- $\log_{1/2} 1/2 = -1$ ($1/2 = 1/a$)
- $\log_2 2 = -1$ ($2 = 1/b$)

$2 = 1/a$

$1/2 = b$

(6) $\log_a 1 = 0$

$a > 0$
$a \neq 1$

PROOF →

$\log_a N = L \iff N = a^L$

$a = a$; $N = 1$

$\log_a 1 = L \iff 1 = a^L$

$L = 0$

Summary →

① $\log_a N_1 + \log_a N_2 = \log_a N_1 N_2$

② $\log_a N_1 - \log_a N_2 = \log_a \frac{N_1}{N_2}$

③ $\log_a a = 1$

④ $\log_a (1/a) = -1$

⑤ $\log_a 1 = 0$

⑥ $\log_a b = \frac{\log_q b}{\log_q a}$

TYPE -1

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Q. Find the value of the following

(i) $\log_{\sin 30^\circ} \cos 60^\circ = ?$

$\log_{\frac{1}{2}} \frac{1}{2} = (1)$ Ans

Q. Find the value of the following

(ii) $\log_{4/3}(1.\bar{3})=?$

$a = \frac{4}{3} = 1.\bar{3}$

$\log_a a = 1$ Ans

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Q. Find the value of the following

(iii) $\log_{10}(\cos 0°) = ?$

$\log_{10} 1 = 0$ Ans

Q. Find the value of the following

(iv) $\log_2(\sin^2 x + \cos^2 x) = ?$

$= 1$

$\log_2 1 = 0$ Ans

Q. Find the value of the following

(v) $\log_{1.5}(0.\bar{6}) = ?$

$a = 1.5 = \frac{3}{2}$

$\frac{1}{a} = \frac{2}{3} = 0.\bar{6}$

$\Rightarrow \log_a \frac{1}{a} = (-1)$ Ans.

Property - 7

★ $\log_a(x^n) = n(\log_a x)$ (Log rule)

PROOF → (Not important)

$\log_a(x^n) = L$ ✓

$N = a^L \Rightarrow x^n = a^L$

Raised power $\left(\frac{1}{n}\right)$

$(x^n)^{1/n} = (a^L)^{1/n}$

$x = a^{L/n}$

$L/\eta = L_1$

$X = a$

N_1

$$L_1 = \log_a N_1$$

$$\frac{L}{\eta} = (\log_a x)$$

$$\Rightarrow L = \eta \cdot (\log_a x)$$

$$= \log_a (x^{\eta})$$

Hence Proved

Modification:

$$\log_a(x^n) = n\cdot(\log_a|x|),\ n \to \text{Even}$$

$$= n(\log_a x),\ n \to \text{odd}$$

ex:- • $\log_a x^4 = 4(\log_a|x|)$

• $\log_a x^3 = 3(\log_a x)$

Q → Find 'x' satisfying

(i) $\log_2(x^4) = 2$

Soln → $4(\log_2|x|) = 2$

$\log_2|x| = \frac{1}{2}$

$|x| = 2^{1/2} = \sqrt{2}$

$|x| = \sqrt{2}$

$x = \pm\sqrt{2}$ Ans

(ii) $\log_2(x^3) = 3$

$3(\log_2 x) = 3$

$\log_2 x = 1$

$x = 2$ Ans

Property - 8

"Stay"

PROOF HW

$$\log_{(a^n)} b = \frac{1}{n} \cdot \log_a b$$

Modification:

$$\log_{(a^n)} b = \frac{1}{n}\left(\log_{|a|} b\right) ; \; n \to \text{even}$$

$$= \frac{1}{n}\left(\log_a b\right) ; \; n \to \text{odd}$$

ex ⟶

$$\log_{a^2} b = \frac{1}{2}\left(\log_{|a|} b\right)$$

$$\log_{a^3} b = \frac{1}{3}\log_a b$$

Q. Find the value of the following

(vi) $\log_{0.125} 8 = ?$

- $0.125 = \frac{1}{8} = (2^{-3})$
- $8 = 2^3$

$$\log_{(2^{-3})} (2)^3 = \frac{3}{(-3)} \cdot (\log_2 2) = (-1) \text{ Ans}$$

Q. Find the value of the following

(vii) $\log_{(2-\sqrt{3})}(2+\sqrt{3}) = ?$

$a = 2 - \sqrt{3}$

$\frac{1}{a} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$

$\log_a \frac{1}{a} = (-1)$ Ans

Q. (i) $\log_{\sqrt{7}}(2401) = ?$

$2401 = (7)^4$

$\sqrt{7} = 7^{1/2}$

$|7| = 7$

$$\log_{(7)^{1/2}} (7)^4 = \frac{4}{(1/2)} (\log_7 |7|)$$

$= 8$ Ans

Q. (ii) $\log_{5\sqrt{5}}(125) = ?$

$125 = 5^3$

$5\sqrt{5} = (5)^{3/2}$

$$\log_{(5)^{3/2}} (5)^3 = \frac{3}{3/2}\left(\log_5 5\right)$$

$= 2$ Ans

Q. (iii) $\log_2\{\log_2\{\log_3(\log_3 27^3)\}\} = ?$

$= \log_3 9 = 2$

(challenge Sawaal) ?

- $\log_3(3^3)^3 = \log_3(3^9)$

$= 9(\log_3 3) = 9$

- $\log_3 \bar{9} = \log_3(3^2) = 2(\log_3 |3|)$

$= 2(\log_3 3) = 2$

- $\log_2(\log_2(\log_2 2))$

$= 1$

$= \log_2 1 = 0$

Ans

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"multiplication-series is given"

TYPE -2

and you have to Find "$\log 1$" $= 0$.

Q. (i) $\log\tan 1^\circ \log\tan 2^\circ \ldots\ldots\ldots\ldots \log\tan 89^\circ = ?$

$= (\log \tan 1^\circ)(\log \tan 2^\circ)\ldots\ldots(\log \tan 45^\circ)\ldots(\log \tan 89^\circ)$

$(\log 1) = 0$

$= 0$ Ans.

Q. (ii) $\log\sin 1° . \log\sin 2° \ldots\ldots\ldots\ldots \log\sin 90° = ?$

$\log 1 = 0$

Ans: "0"

PROPERTY-9 ⇒

$$a^{(\log_a N)} = N$$

same

ex ↦

- $2^{(\log_2 8)} = 8$
- $3^{(\log_3 27)} = 27$

PROOF → LHS ⇒

assume $\log_a N = L \Rightarrow \boxed{N = a^L}$

$$\text{LHS} = a^L = N = \text{RHS}$$

Hence PROVED.

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PROPERTY-10

$$a^{(\log_b c)} = c^{(\log_b a)}$$

★ PROOF HW

"MSM"

Examples →

- $2^{\log_3 4} = 4^{\log_3 2}$ ✓
- $3^{\log_{10} 7} = 7^{\log_{10} 3}$ ✓

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"JBK... JBK... JBK"

Revise complete Notes "Pen and Paper" write in Flash-pad.

HOMEWORK SLIDE

DPP 6 : ~~1,2,3,4,5,6~~,7,(8),(9)

acchesawaal

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SET THEORY LEC-1

Next turn →

First Hour → Doubts, Log PYQ and more Practice

Last Hour → Set theory

JBK...JBK...JBK

- TO *FUTURE IIT-IANS*